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Shocks in groundwater resource management

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Abstract

We consider the case of two exogeneous and deterministic shocks in groundwater resource management: an increase in extraction costs and a decrease in the recharge rate. Both shocks may be a consequence of climate change. We show that the optimal adaptation behaviour to these shocks is non-monotonic and goes in opposite direction in the long-run. In the short run, both shocks lead to compensation-seeking behaviour and thus to quicker extraction, although we do not consider uncertainty. We show that this compensation-seeking behaviour is linked to the pumping cost externality: the greater this externality the more extreme the extraction behaviour in the short-run.

JEL classification: C61,Q25.
Key words: Groundwater resource, optimal behavior, exogeneous shocks, jumps.

1 Introduction

Over the last years, the concern about the future availability of water has increased, caused largely by the phenomenon of global warming. We are in particular interested in the successive episodes of droughts of the last years, which led to a critical situation in the farming sector (see Amigues[1]). The phenomenon of drought decomposes into three different events: the meteorological drought which corresponds to a lack of precipitation, the hydrological drought which takes place when the available water supply in groundwaters, lakes and reservoirs falls below the average, and the agricultural drought which occurs when there is not enough humidity for the crops, even if precipitations are normal. These periods of drought can trigger structural problems of discrepancy between needs and water ressources. In this context, our study concerns the problem of insufficient filling of reservoirs, or groundwater resources, linked to the hydrological drought. In particular, we

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are interested in determining the optimal behavior of the farmer, the user of the water resource, in the face of this drought.

We are especially interested in the use of dynamic modelling to find the optimal path of water extraction, (see Gisser and Sanchez [5]). Gisser and Sanchez [5] present a model of groundwater extraction in which they analyze the difference between two strategies: the optimal solution and the competitive case. They conclude that if the capacity of the aquifere is big, the difference between both stategies is unimportant. On the other hand, numerous papers studied the optimal path of extraction by incorporating an externality: the contamination of the aquifer. In this context, papers as Cummings [2], Dinar and Xepapadeas [4], Tsur and Zemel [14], Moreaux and Reynaud [9], deal with optimization models for groundwater management under conditions of saltwater intrusion and, Zeitouni and Dinar [15] compare two situations: the optimal joint pumping management, in which two adjacent aquifers of different water qualities are interrelated and the independent aquifer pumping. Finally, optimal management of quantity and quality of a common groundwater resource is studied in general in Roseta-Palma, [11] and [12]. In our case, we do not model an additional externality, such as contamination. We disturb the dynamic of the system by introducing the occurrence of a drought. To do so, we use a simple groundwater model, as those developed in Gisser and Sanchez [5] and Roseta-Palma [11], in which we analyse the impact of shocks on the optimal path of extraction.

To introduce shocks into our initial model, we also refer to the literature on shocks, or on the arrival of an event, in the management of a resource, as Long [7] and Tsur and Zemel [13]. For example, in [7], Long analyzes a model of extraction of a non-renewable resource in which there is a finite time until the uncertain arrival of a nationalization. In case of sure nationalization, he concludes that the optimal behavior is the exhaustion of the resource in finite time. Tsur and Zemel [13] deal with the management of groundwater under threats of catastrophic events. They distinguish endogenous and exogenous events. Endogenous events happen if a critical stock level is reached. Concerning endogeneous events, they show that if the event is certain, the optimal behavior is not to arrive at the critical level and if it is uncertain, it is optimal to increase extractions. With regards to exogeneous events, the event can be irreversible or reversible depending on how it increases costs. In our study, we will introduce two exogenous and reversible shocks which may occur because of the lack of water. We prove that, even in a simple case of drought, where the event is deterministic, exogenous and reversible, the optimal behavior consists in the encouragement of extractions before the event, to compensate for future losses. We can find this result in the existing literature in another context. In fact, in [8], Long and Sinn analyse the idea that an increase in the price of oil will stimulate the extraction of oil at the time of announcement and, in a recent paper, [3], Di Maria, Smulders and Van Der Werf find that, when allowing some time between announcement and implementation of a cap on carbon dioxide emissions, emissions from non-renewable energy sources increase at the time of announcement.

This paper is organized in the following way. In section 2, we present a model of extraction of groundwater at infinite time. We then introduce two exogenous shocks which incorporate the idea of the lack of water. In section 3, we make a numeric illustration where we analyze the optimal behavior in the short and long-run, as well as the impact of the various costs of the model on the optimal behavior. Finally, in section 4, we conclude in give some perspectives for future research.

2 The model

2.1 A simple groundwater model

We base our analysis on a simple groundwater extraction model (see for example Gisser and Sanchez [5], Tsur and Zemel [14], Roseta-Palma 2002 [11] among others). Consider a groundwater stock, $G(t)$ with a constant recharge rate r which is used for agricultural production. The groundwater stock evolves according to the following equation[1]:

$$\dot{G} = -g + r$$

where $g(t)$ the amount of water pumped. As in Roseta-Palma, [11],[12] let $y(g(t))$ be the agricultural production function and p_y the unit price of production. In contrast to Roseta-Palma, consider that there are two types of costs: extraction costs $c_e(G, g)$, to pump the water to the surface, and conveyance costs, $c_c(g)$ to bring the water from the well to the field. Moreover, assume extraction costs of the form: $c(G)g$, with $c(G)$ the decreasing unit cost of extraction. The instantaneous net benefits from agricultural production then amount to

$$p_y y(g) - c(G)g - c_c(g).$$

The problem is to choose the optimal extraction path such as to maximise the present value of net benefits, with ρ the discount rate, taking into account the evolution of the resource stock and its impact on net benefits:

$$\max_{g(.)} \int_0^{\infty} \quad [p_y y(g) - c(G)g - c_c(g)]\mathrm{e}^{-\rho \mathrm{t}} \quad \mathrm{dt} \tag{1}$$

$$s.c \qquad \dot{G} = -g + r \tag{2}$$

$$G(0) = G_0 \quad \text{given,} \tag{3}$$

$$\text{and} \quad g \geq 0 \qquad G \geq 0. \tag{4}$$

[1] We omit the time indicator in all following equations, whenever this is possible without causing misunderstandings, in order to make equations more easily readable.

The resource stock at the beginning of the problem is known (equation 3). The extraction volume and the resource stock cannot become negative, as state conditions (4). The Hamiltonian of this problem is given by:

$$H = p_y y(g) - c(G)g - c_c(g) + \lambda(-g + r), \tag{5}$$

where λ is the adjoint variable. Applying the maximum principle and supposing interior solutions, we have the usual first order conditions:

$$\frac{\partial H}{\partial g} = 0 \quad \Rightarrow \quad p_y \frac{\partial y}{\partial g} = c(G) + \frac{\partial c_c}{\partial g} + \lambda, \tag{6}$$

$$\dot{\lambda} = -\frac{\partial H}{\partial G} + \rho\lambda \quad \Rightarrow \quad \dot{\lambda} = \frac{\partial c}{\partial G} g + \rho\lambda. \tag{7}$$

Equation (6) states that marginal benefits from agricultural production equal marginal costs, decomposed in marginal extraction costs, marginal conveyance costs and the cost of extraction today, rather than tomorrow, expressed in the shadow price, λ. Equation (7) states that the growth of the shadow price depends on the opportunity cost expressed in the discount rate and the stock effect, which describes the impact of extraction on unit extraction costs.

For some functional forms, the optimal extraction paths, $G(t)^*$ $g(t)^*$ and $\lambda(t)^*$ can be solved for analytically, as we remember in the Appendix (A.1.1).

2.2 Two exogeneous shocks

In this section, we are going to add two exogenous shocks to our initial problem,(see section 1.1). Both shocks represent two direct consequences of a drought. On the one hand, we are going to model an increase in the costs of extractions at the known moment ta. We assume that the government implements a law that increases the costs of extraction of the common groundwater resource because of a predictable drought. This measure will take place from the moment ta on (given). On the other hand, we are going to model a decrease of the recharge rate at the known moment ta. We assume indeed that mean precipitations decrease in the future. Then, less water will arrive in the groundwater resource.

2.2.1 A raise in extraction costs

The problem now becomes:

$$\max_{g(.)} \int_0^\infty \quad [p_y y(g) - c_e(G, g) - c_c(g)]\mathrm{e}^{-\rho \mathrm{t}} \quad \mathrm{dt} \tag{8}$$

$$c_e(G, g) = \begin{cases} c(G)g & \text{if} \quad t \leq t_a \\ \bar{c}(G)g & \text{if} \quad t > t_a \end{cases} \tag{9}$$

$$\dot{G} = -g + r \tag{10}$$

$$G(0) = G_0, \quad \text{and} \quad \bar{c} > c, t_a \quad \text{given}, \tag{11}$$

$$g \geq 0 \qquad G \geq 0. \tag{12}$$

We can solve the problem in two steps. First, we have to find $\phi(ta, G_{ta})$, the scrap value function that represent the maximisation between t_a and ∞, that is:

$$\phi(ta, G_{ta}) = \max_{g(.)} \int_{t_a}^{\infty} \qquad [p_y y(g) - c_e(G, g) - c_c(g)]\mathrm{e}^{-\rho \mathrm{t}}\mathrm{dt}.$$

After this, our problem is to find G,g and $G(t_a)$ that maximise:

$$\int_0^{t_a} \qquad [p_y y(g) - c_e(G, g) - c_c(g)]\mathrm{e}^{-\rho \mathrm{t}}\mathrm{dt} + \phi(\mathrm{t_a}, \mathrm{G_{ta}}),$$

with the former conditions. As before, we may write the Hamiltonian of this problem:

$$H = p_y y(g) - c(G)g - c_c(g) + \pi(-g + r). \tag{13}$$

where π is the adjoint variable. We are now in a free-endpoint problem, with t_a known and need an additional transversality condition (see for example Léonard and Ngo van Long [6]):

$$\pi(t_a) = \frac{\partial \phi(ta, G_{ta})}{\partial G_{ta}}. \tag{14}$$

The full resolution of this modified extraction problem is deferred to the Appendix, (A.1.2).

2.2.2 A decrease in the recharge rate

We therefore assume that, from t_a on, the recharge rate is reduced from r_1 to r_2. The problem becomes:

$$\max_{g(.)} \int_0^{\infty} \qquad [p_y y(g) - c(G)g - c_c(g)]\mathrm{e}^{-\rho \mathrm{t}} \quad \mathrm{dt} \tag{15}$$

$$\dot{G}(t) = \begin{cases} -g_t + r_1 & \text{if} \qquad t \leq t_a \\ -g_t + r_2 & \text{if} \qquad t > t_a \end{cases} \tag{16}$$

$$G(0) = G_0, t_a \quad \text{given}, \qquad r_1 > r_2, \tag{17}$$

$$g \geq 0 \qquad G \geq 0. \tag{18}$$

We solve this problem in the same way as the previous one.

3 An illustration

3.1 A numerical example

To illustrate graphically the impact of the two exogeneous shocks on optimal extraction decisions, we propose the following simple example. We suppose agricultural production is linear in water input. Unit extraction costs are a linear and decreasing function of the water stock: with the depletion of the groundwater stock, the extraction of one additional cubic meter of water is more and more expensive. Conveyence costs are an increasing and quadratic function of the water volume used: water transport becomes more and more complicated the further away the irrigated fields, for example because of physical obstacles. Consider again the following functional forms which we proposed for the analytical solutions of the previous sections:

$$y(g) = bg\gamma, \qquad c_e(G,g) = (Z - CG)g, \qquad c_c(g) = \frac{vg^2}{2},$$

where b is the productivity coefficient, γ are inputs other than water, for example fertilizer (see Roseta-Palma [11]). Z represents the unit extraction costs for the last unit of water on the bottom of the groundwater resource. C describes the speed at which cost raise with decreasing stock levels. v is a conveyance cost parameter. b,γ, Z, C and v are positive and given parameters. We also have to verify that $Z - CG > 0$ for all $G(t)$.

Let's consider the following numerical values:

$$\rho = 0.05, p = 10, \gamma = 0.2, b = 0.8, Z = 2, C = 0.02, v = 0.8, G_0 = 100, r = 0.5$$

for the simple problem and:

$$r_1 = 0.5, \quad r_2 = 0.25, \quad B = 0.005$$

for the two problems with shocks[2].

In the case of the first shock 2.2.1, the cost-increase is written as:

$$\bar{c}(G) = c(G) + B = Z - CG + B.$$

In the case of the shock 2.2.2, the recharge rate is reduced from $r_1 = 0.5$ to $r_2 = 0.25$.

Note that when the resource is at its initial level, $G_0 = 100$, the extraction cost is: $Z - CG = 2 - 2 = 0$.

Figure 1 indicates the shape of the optimal extraction paths over time for the simple extraction problem described in section 1. Both the optimal groundwater stock and the optimal extraction volume are decreasing and approach the steady state asymptotically. Let us now discuss the impact of the two exogeneous shocks.

[2]To fit the values to the reality, we measure volumes of water in thousand of m^3 and prices in hundreds of euros, for example the initial resource stock value is $100 * 10^3 = 100000m^3$ and the cost of one ton of agricultural production, p, is $10 * 10^2 = 1000$ euros.

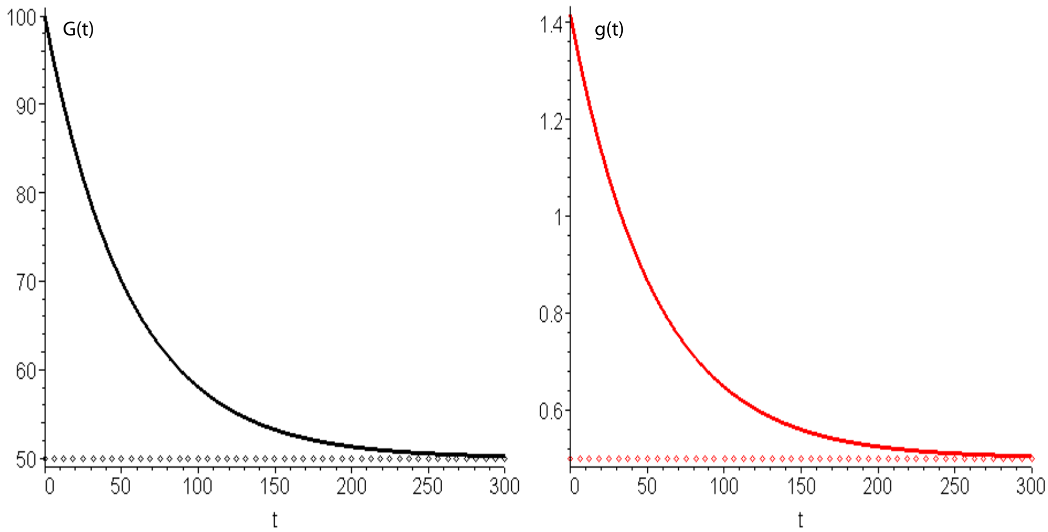

Figure 1: Optimal evolution of stock $G(t)$ and extraction $g(t)$ in the simple problem

3.2 Optimal policy with shocks at the steady state

Figure 2 represents optimal extraction paths in the simple problem (line) and after a raise in costs (diamonds). In the long run, extraction is more conservative after the shock: the steady state stock is greater than before. This is because increased extraction costs render extraction non-profitable from an earlier point in time on.

Figure 3 represents optimal extraction paths in the simple problem (line) and after a decrease in recharge rates (diamonds). In the long run, extraction is less conservative: the steady-state stock is smaller than before. This is because the recharge flow is slower in the future. It is thus necessary to deplete the resource stock further to compensate for this decrease.

We can also confirm the results observed in the above figures by comparing the steady-steate stock of the simple problem (SP) with the steady-state stock after the two shocks (EC: Increase in extraction costs and, DR: Decrease in recharge rate), with the following equations:

$$GSP_\infty = \frac{rv}{C} + \frac{r}{\rho} - \frac{p_y b\gamma}{C} + \frac{Z}{C}$$

$$GEC_\infty = \frac{rv}{C} + \frac{r}{\rho} - \frac{p_y b\gamma}{C} + \frac{Z}{C} + \frac{B}{C}$$

$$GDR_\infty = \frac{r_2 v}{C} + \frac{r_2}{\rho} - \frac{p_y b\gamma}{C} + \frac{Z}{C}$$

We see that,

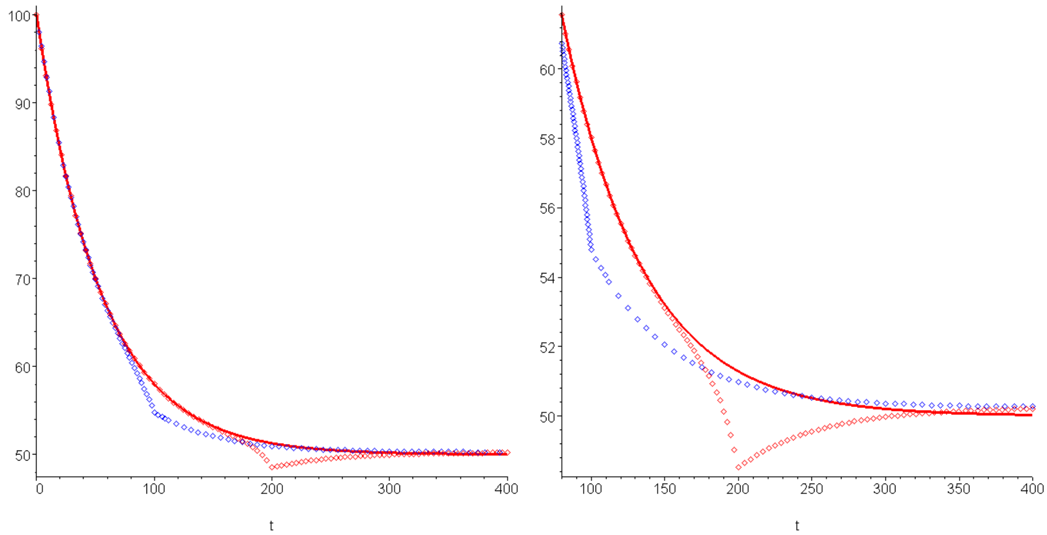

Figure 2: Raise in costs. Optimal evolution of stock $G(t)$. Red line: simple problem. Diamonds: problem after raise in costs in ta=100 and ta=200. Right-hand side: zoom on periods where shocks occur.

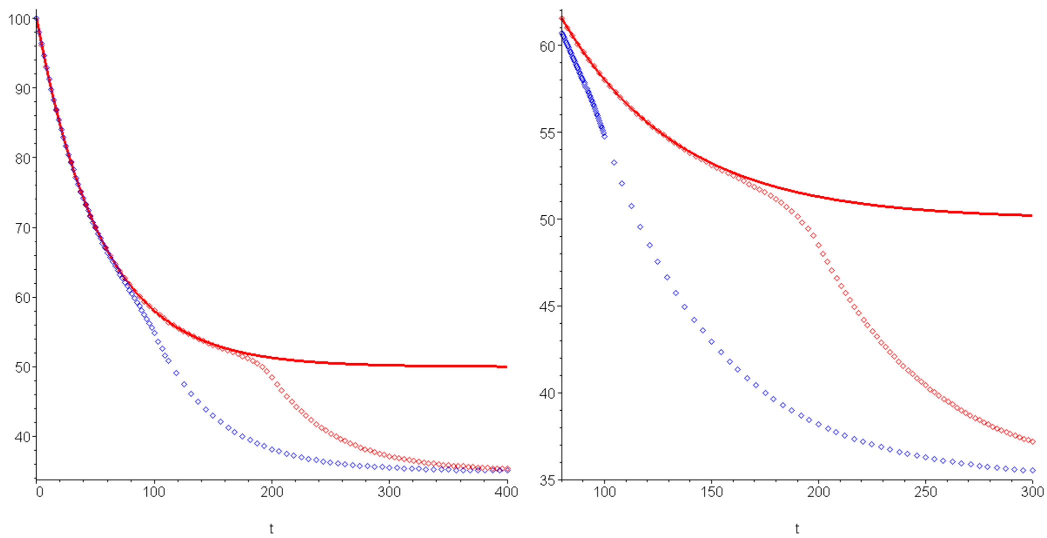

Figure 3: Decrease in recharge rate. Optimal evolution of stock $G(t)$. Red line: simple problem, diamonds: problem after decrease in recharge rate in ta=100 and ta=200. Right-hand side: zoom on periods where shocks occur.

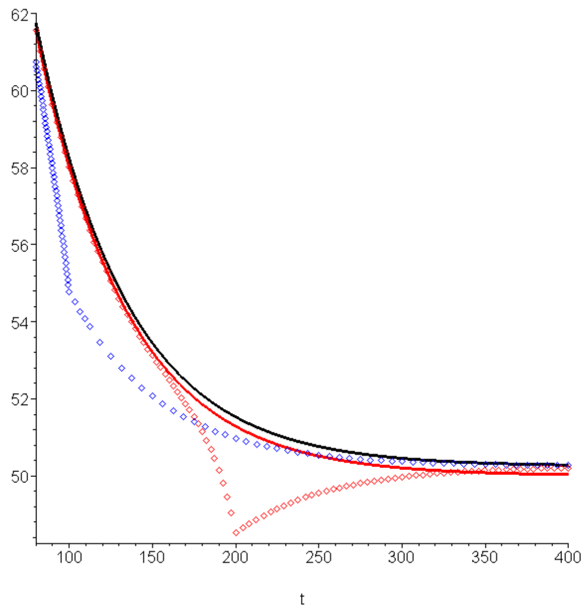

Figure 4: Raise in costs from t=0 on. Optimal evolution of stock $G(t)$. Red line: simple problem, black line above: problem with raised costs from t=0 on. Diamonds: problem after raise in costs in ta=100 and ta=200

$$GEC_\infty = GSP_\infty + \frac{B}{C} > GSP_\infty > GDR_\infty, \quad \text{because} \quad r_2 < r_1 = r$$

In both cases, the steady state equilibrium corresponds to the one which would occur when the shock was in place from the very beginning of the problem. Indeed, in figure 4, we have represented the case where the cost-increase is effective from $ta = 0$ on (black line). In this case, higher extraction costs make extraction slow down earlier than in the simple case and the long-term equilibrium of resource stocks is higher than before, as in the long-run equilibrium after the shock. Likewise, in figure 5, we have plotted the case where the recharge rate has decreased from $ta = 0$ on (black line). The optimal extraction path lays underneath the extraction path of the simple problem (red line). This is where the optimal extraction paths after the recharge shock (diamonds) tends towards.

3.3 Optimal policy with shocks during the transition path

Both exogenous shocks lead to a more intense extraction in the short run. This is to compensate a loss that is due to the shocks. Indeed, if we compute the sum of payoffs from ta on, we find that they are smaller for the two examples with shocks than for the simple problem (see table (1)).

The later the shock arrives, the greater the compensation that is possible. Indeed, let's

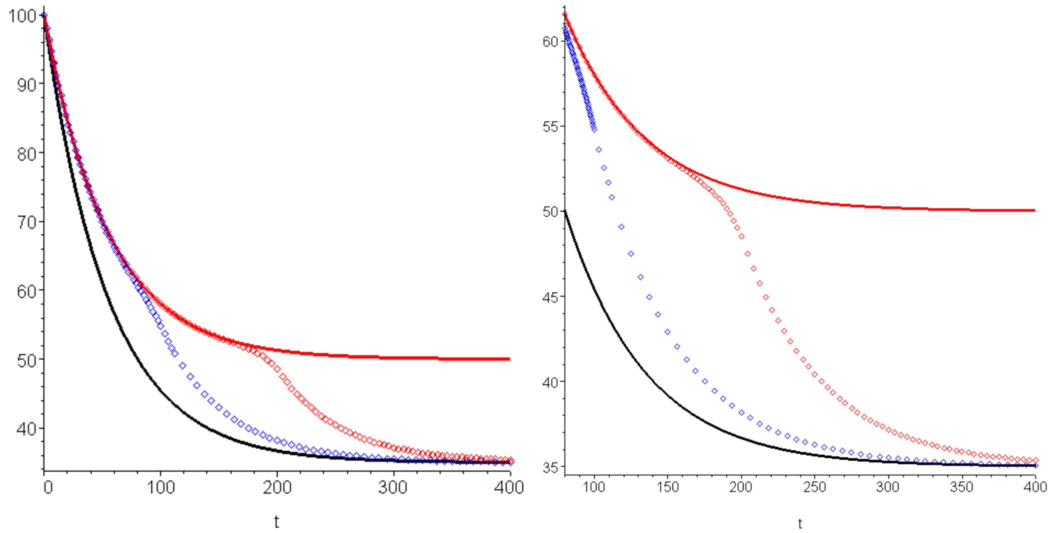

Figure 5: Decrease in recharge rate from t=0 on. Optimal evolution of stock $G(t)$. Red line: simple problem, black line below: problem with decreased recharge from t=0 on. Diamonds: problem after decrease in recharge rate in ta=100 and ta=200. Right hand side: zoom on periods where shocks occur.

Table 1: Payoffs from ta on

	$t_a = 0$	$t_a = 100$	$t_a = 200$
$P_{ta}SP$	20.69	0.0389	0.00019
$P_{ta}EC$	20.58	0.0334	0.00016
$P_{ta}DR$	18.82	0.0271	0.00012
$P_{ta}SP - P_{ta}EC$	>0	>0	>0
$P_{ta}SP - P_{ta}DR$	>0	>0	>0

SP: Simple problem

EC: Raise in extractions costs

DR: Decrease of the recharge rate

turn back to figure 2: when the shock occurs in $ta = 100$ the optimal path has a small peak but when it occurs in $ta = 200$, there is a big peak. The non-smooth extraction behavior in the short run may also be explained by the trade-off between two extraction paths. Reconsider figure 5: the optimal extraction in case of a recharge rate decrease (diamonds) follows the path of the simple problem (red line) in the first place, but then adjusts to the path that would have been optimal if the recharge was reduced from $t = 0$ on, afterwards (black line).

3.4 The impact of costs on the optimal behavior

The shape of the optimal paths is also influenced by the cost structure of our problem. Let's assume the time of occurence of the shock is fixed: $ta = 100$.

3.4.1 A change in extraction costs

First, we are interested in examining the impact of a change in extraction costs in case of shock 2.2.1. In figure 6, we have depicted the optimal evolution of the stock (left-hand side) and optimal extraction volumes (right-hand side) for three different extraction costs: $Z = 6$ and $C = 0.06$ on top (black diamonds), $Z = 2$ and $C = 0.02$ in the middle (blue diamonds, the reference case) and $Z = 1.1$ and $C = 0.011$ on the bottom (red diamonds)[3]. We can see that the peak $(G_\infty - G_{ta})$ and the jump $(g_\infty - g_{ta})$ from one extraction behavior to another, are more important the greater the extraction costs. Inversely, the smaller the extraction costs, the less important the peak and the jump.
Note that increasing (decreasing) extraction costs also trigger an increase (decrease) in the long-term steady state stock. Hence, the loss to be compensated by the shock is greater, which explains the bigger jump in extraction behavior (or the bigger peak in the optimal evolution of the stock).

We also studied the impact of the change of extraction costs in case of shock 2.2.2. We can see the results in figure 7. The optimal behavior is similar to that in the case 2.2.1: in the long run, the stationary level of the resource increases (or decreases), when extraction cost increases (decreases). The greater the pumping cost, the less groundwater is used and the more water is left in the acquifer. In the short run, the peak in G_{ta} (or the jump in g_{ta}) becomes more important with an increase in extraction costs. Thus, the greater the peak (or the jump), the more water is pumped before the arrival of the shock to compensate for the losses after the shock (as in section, 3.3).

3.4.2 A change in conveyence costs

Next, look at the impact of conveyence costs on the optimal extraction behavior of the shock 2.2.1. In figure 8, we have increased conveyence costs from $v = 0.8$ (blue diamonds,

[3]We have changed C and Z such that the maximum stock level is unchanged.

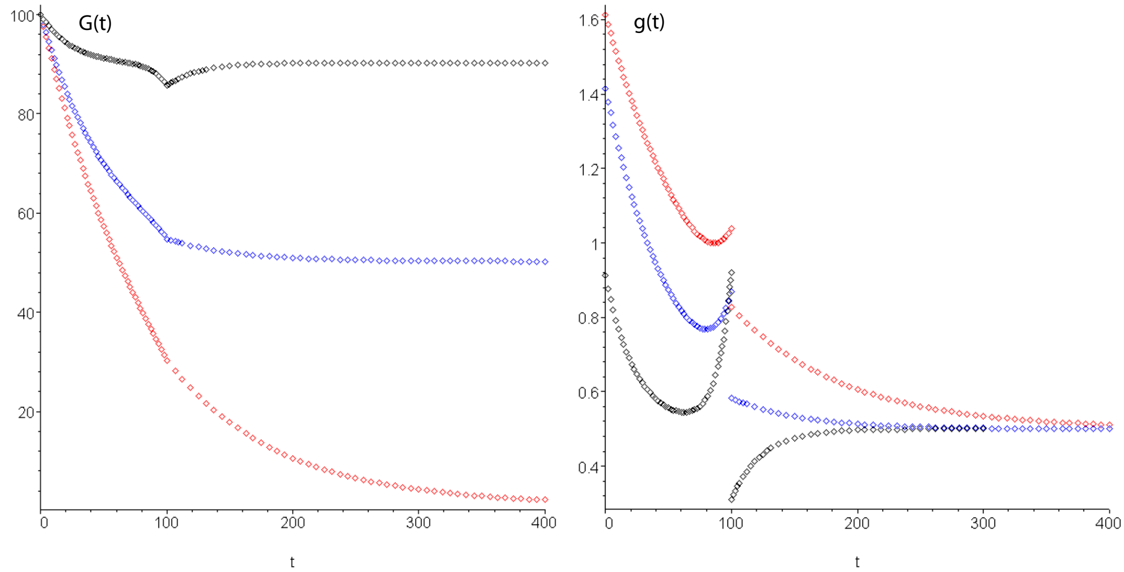

Figure 6: Optimal evolution of $G(t)$(left-side) and $g(t)$(right-side) for the shock 2.2.1 in $ta = 100$, for different extraction costs. Blue diamonds: reference case, $Z = 2$ and $C = 0.02$. Black diamonds: cost increase, $Z = 6$ and $C = 0.06$. Red diamonds: cost decrease, $Z = 1.1$ and $C = 0.011$.

reference case) to $v = 2$ (black diamonds) and decreased them from $v = 0.8$ to $v = 0.3$(red diamonds).

We can see in figure 8 and figure 9 that increasing (decreasing) conveyence costs, decrease(increase) the jump in the decision variable and decrease(increase) the peak in the state variable for the two shocks.

3.4.3 Comparison of the impact of both costs

Hence, the higher extraction costs, the higher the peak and the jump in optimal behavior. On the other hand, the higher conveyence costs, the smaller the peak and the jump. We can see the results in table 2. These results are not independent because the relative weight of both costs plays a role.[4] The "compensation seeking" behavior which leads to peaks in the optimal paths is thus first and foremost linked to the pumping cost externality (see Provencher and Burt 1993 [10]). The greater this externality, the more extreme the short run reaction to exogeneous shocks.

In order to better understand the influence of the different costs on the solution of our

[4] We have also confirmed that in a problem without extraction costs, there is no peak anymore. Inversly, if v is absent, we have a linear extraction problem with a peak.

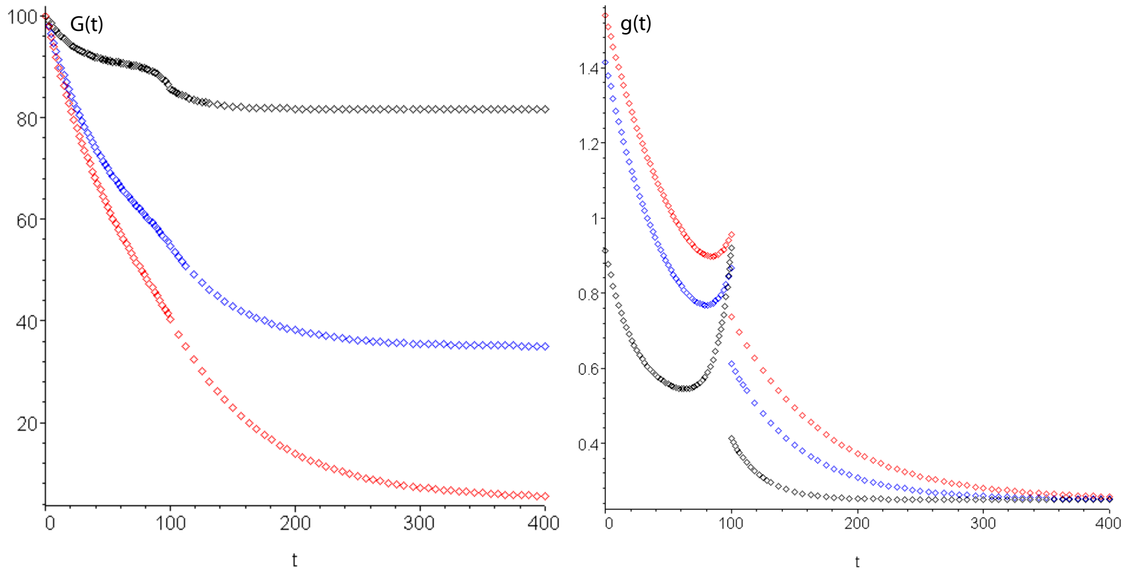

Figure 7: Optimal evolution of extraction $G(t)$(left-side) and $g(t)$(right-side) for the shock 2.2.2, in $ta = 100$, for different extraction costs. Blue diamonds: reference case, $Z = 2$ and $C = 0.02$. Black diamonds: cost increase, $Z = 6$ and $C = 0.06$. Red diamonds: cost decrease, $Z = 1.4$ and $C = 0.014$.

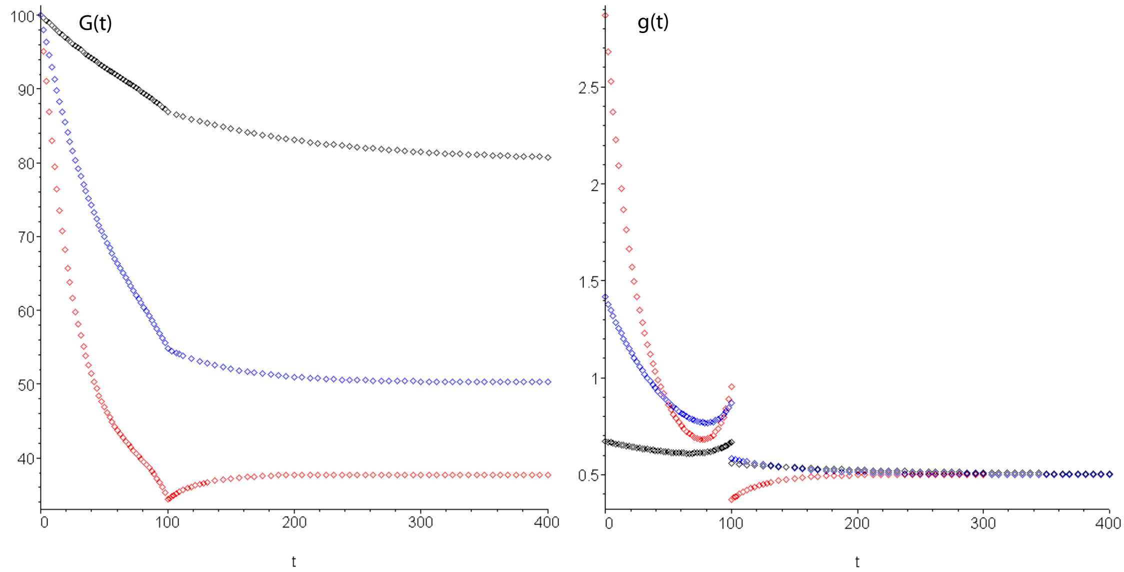

Figure 8: Optimal evolution of extraction $G(t)$(left-side) and $g(t)$(right-side) for the shock 2.2.1 in $ta = 100$, for different conveyence costs. Blue diamonds: reference case, $v = 0.8$. Diamonds: increase in conveyence costs, $v = 2$ (blue) and decrease in conveyence costs,$v = 0.3$ (red).

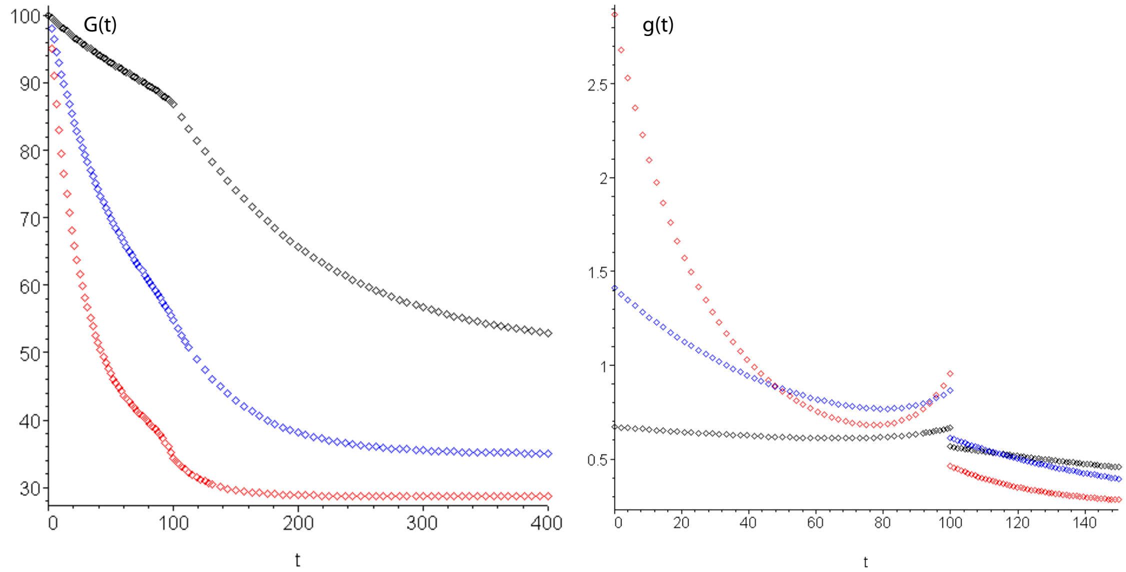

Figure 9: Optimal evolution of extraction $G(t)$(left-side) and $g(t)$(right-side) for the shock 2.2.2 in $ta = 100$, for different conveyence costs. Blue diamonds: reference case, $v = 0.8$. Diamonds: increase in conveyence costs, $v = 2$ (blue) and decrease in conveyence costs,$v = 0.3$ (red).

problem, we decided to analyze numerically the variation of the peak($G_\infty - G_{ta}$) in shock 2.2.1 as a function of the parameters of the model.

In figure 10, we can observe that the variation of the peak with regards to the parameter v is not monotonous. Moreover, the peak is positive when v is situated between 0 and 0.5 and when v is greater than 3. We can also show the influence of parameters (Z, C). The greater (Z, C), the more the peak increases. That is, the greater extraction costs, the more short-run pumping will occur (as before). The peak is positive when (Z, C) are greater than 0.025.

In figure 11, we studied the impact of the moment of occurence of the shock (ta) and of an increase in costs (B). As expected, the later the event takes place (ta big) and the more important the increase of costs (B big), the greater the difference between G_∞ (stationary state of the resource) and G_{ta} .

Note: An increase of the parameters (Z, C) and of the parameter B has the same effect on the peak, because in both cases, we have an increase in extraction costs.

Likewise, we have analyzed the variation of the peak in problem 2.2.2. We can observe in figure 12 that the optimal behavior is similar to the one described with the previous shock. The peak is positive from $C = 1.5$. On the other hand, there is a significant difference between both shocks: the peak is negative over a much bigger interval, when v is

	G_{inf}	Pic= G_{inf} - G_{ta}	Saut = g_{ta}^{+} - g_{ta}^{-}
Z, C ↑	↑	↑	↑
Z, C ↓	↓	↓	↓
V ↑	↑	↓	↓
V ↓	↓	↑	↑

Table 2: Results for the impact of costs on the optimal behavior.

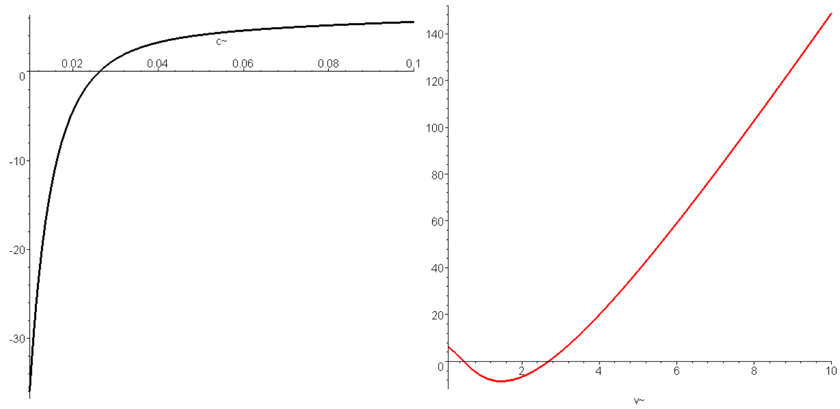

Figure 10: G_∞ - G_{ta} according to the parameters bound to the costs(v and (Z, C)) in the shock 2.2.1.

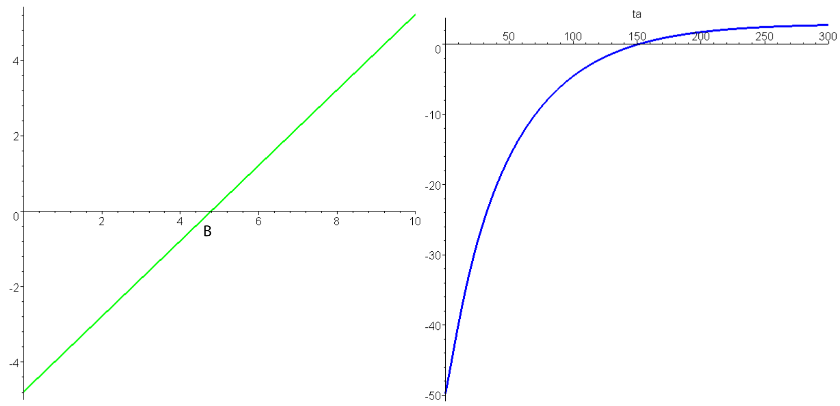

Figure 11: G_∞ - G_{ta} according to the parameters ta and B in 2.2.1.

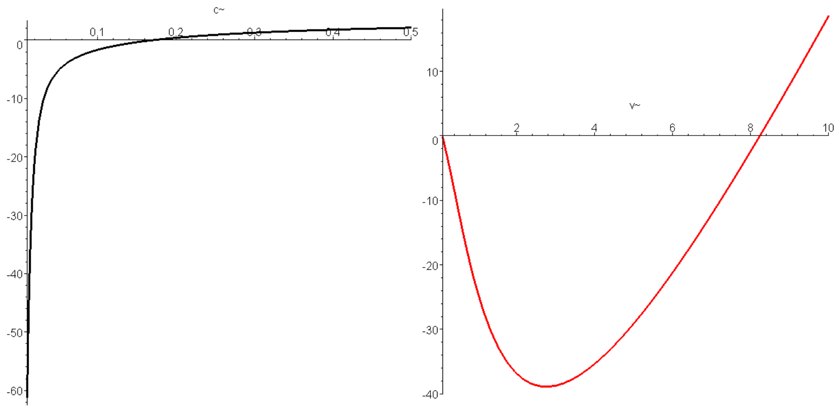

Figure 12: G_{inf} - G_{ta} according to the parameters bound to the costs(v and (Z, C)), in the shock 2.2.2

between 0 and 8.

The interest to study the positivity and the negativity of the peak is linked to the importance of the short-term reaction. When the peak is positive (negative), the pumping before ta (i.e. the compensation of the losses) is more (less) important.

4 Conclusion

In this paper, we presented the analytical solutions of an optimization problem with infinite time horizon, dealing with the extraction of a renewable groundwater resource. We studied the impact of two exogenous shocks on the resource: the increase in extraction costs and the decrease of the recharge rate. By modelling these two shocks, our purpose was to represent two different consequences of a drought.

In the long run, we found a difference between the two shocks. One is more conservative than the other, independently of the values of parameters chosen. We proved that in the case of the increase in extraction costs, the stationnary level of the resource is situated above the steady-state of the simple problem. On the other hand, in the case of the decrease of the recharge rate, the steady-state is below the steady-state of the simple problem. We thus confirm, logically, that the law imposed by the government, that is, the increase of the costs of extractions, is the more conservative solution in the long term.

In the short run, the optimal extraction behavior in the face of shocks is the same: extractions are encouraged before the arrival of the shock, to compensate for future losses, and extractions decrease after the shock, to restore the resource. We remind that in [13], Tsur and Zemel [13] obtain the same conclusion in the case of endogenous and uncertains events, that is, the encouragement of extractions before the arrival of the event. However, in contrast to Tsur and Zemel[13], we considered naturally reversibles shocks for the resource. Furthermore, we proved that the impact of the shock on the optimal extraction behavior also depends on the other parameters as the moment of occurrence of the shock or the type of extraction costs.

There are various possible extensions to our paper. We could render the occurence date or the size of the exogeneous shock uncertain. It would then be interesting to compare the outcome of this problem with the results obtained in the literature, (see Tsur and Zemel [13]). We could also study the impact of several successive changes in recharge rates or costs: for example recharge could decrease because of decreased precipitations, as discussed above, but it could then increase again, for example as a result of investments in desalinisation plants. Finally, uncertainty about the extend of climate change may diminish over time: if new information is acquired during the considered time period, we would need a new optimization method, taking into account rolling horizons.

A Optimal extraction paths

A.1 A linear quadratic case

Consider the following functional forms:

$$y(g) = bg\gamma, \qquad c_1(G, g) = (Z - CG)g, \qquad c_2(g) = \frac{vg^2}{2}, \tag{19}$$

where b, γ, Z, C and v are positive given parameters. Thus:

$$\frac{\partial y}{\partial g} = b\gamma > 0, \qquad \frac{\partial c_1}{\partial g} = c(G) = Z - CG, \qquad \frac{\partial c_1}{\partial G} = -C < 0, \qquad \frac{\partial c_2}{\partial g} = vg.$$

A.1.1 Resolution of the simple extraction problem

To solve problem (1)) to (4), with the above functional forms (19), we proceed as follows: Substituting (19) into (6) and (7) and rearranging, we find the optimal extraction volume as a function of the resource stock and the shadow price:

$$g = \frac{p_y b\gamma - Z + CG - \lambda}{v}. \tag{20}$$

Substituting (20) into the equations of motion of the state and adjoint variable (2) and (7), we have the following dynamic system:

$$\dot{G} = C1 - \frac{C}{v}G + \frac{1}{v}\lambda,$$

$$\dot{\lambda} = C2 - \frac{C^2}{v}G + (\frac{C}{v} + \rho)\lambda,$$

with $C1$ and $C2$ constants, and $G(0) = G_0$, which allows us to find the roots of the characteristic polynom:

$$\rho_{1,2} = \frac{\rho \pm \sqrt{\rho^2 + 4\frac{C\rho}{v}}}{2}.$$

We can also find the steady state of the system, for $\dot{G} = 0$ and $\dot{\lambda} = 0$:

$$G_\infty = \frac{rv}{C} + \frac{r}{\rho} - \frac{p_y b\gamma}{C} + \frac{Z}{C}, \tag{21}$$

$$\lambda_\infty = Cr/\rho. \tag{22}$$

Equation (21) results from substitution of (20) into the equation of motion of the state variable (2) and equation (22) results from substitution of (20) and (6) into (7).
Finally, we have the optimal extraction paths, with ρ_2, the negatif root:

$$G(t) = \mathrm{e}^{\rho_2 \mathrm{t}}(\mathrm{G}_0 - \mathrm{G}_\infty) + \mathrm{G}_\infty, \tag{23}$$

and

$$\lambda(t) = \mathrm{e}^{\rho_2 \mathrm{t}}(\lambda_0 - \frac{\mathrm{Cr}}{\rho}) + \frac{\mathrm{Cr}}{\rho}, \tag{24}$$

$$\lambda_0 = p_y b\gamma - Z + CG_0 - v(r - \rho_2(G_0 - G_\infty)),$$

which we find with (2) and (6).

A.1.2 Resolution of the modified extraction problem

Consider here problem (8) to (12), with the functional forms (19). To solve this problem, we will separate it in two parts and proceed by backward induction. First, we solve the maximisation between t_a and infinity, proceeding as in (A.1.1).

We have the functions:

$$G(t) = \mathrm{e}^{\rho_2(\mathrm{t-ta})}(\mathrm{G_{ta}} - \mathrm{G}_\infty) + \mathrm{G}_\infty, \tag{25}$$

$$\lambda(t) = \mathrm{e}^{\rho_2(\mathrm{t-ta})}(\lambda_{\mathrm{ta}} - \frac{\mathrm{Cr_2}}{\rho}) + \frac{\mathrm{Cr_2}}{\rho}, \tag{26}$$

$$g(t) = k1 - k2\mathrm{e}^{\rho_2(\mathrm{t-ta})}, \tag{27}$$

with,

$$k1 = r_2,$$

$$k2 = \rho_2(G_{ta} - G_\infty),$$

and,

$$G_\infty = \frac{r_2 v}{C} + \frac{r_2}{\rho} - \frac{p_y b\gamma}{C} + \frac{Z}{C}, \tag{28}$$

$$\lambda_{ta} = p_y b\gamma - Z + CG_{ta} - vr_2 + v\rho_2(G_{ta} - G_\infty). \tag{29}$$

Substituting (25),(26) and (27) into the objective function, we can compute the scrap value: $\phi(ta, G_{ta})$.

We can now turn to the second part of the problem, between 0 et t_a, considering the optimal solution of the first part. We know that the solutions of the problem are of the shape:

$$G(t) = \bar{A}\mathrm{e}^{\rho_1 \mathrm{t}} + \bar{\mathrm{B}}\mathrm{e}^{\rho_2 \mathrm{t}} + \bar{\mathrm{C}},$$

$$\lambda(t) = \bar{D}\mathrm{e}^{\rho_1 \mathrm{t}} + \bar{\mathrm{E}}\mathrm{e}^{\rho_2 \mathrm{t}} + \bar{\mathrm{F}}.$$

We have (10),(7) and the conditions

$$G(0) = \bar{A} + \bar{B} + \bar{C} = G_0, \tag{30}$$

$$\pi(t_a) = \bar{D}\mathrm{e}^{\rho_1 \mathrm{ta}} + \bar{\mathrm{E}}\mathrm{e}^{\rho_2 \mathrm{ta}} + \bar{\mathrm{F}} = \pi_{\mathrm{a}}, \tag{31}$$

$$\pi(t_a) = \frac{\delta\phi(ta, G_{ta})}{\delta G_{ta}}, \tag{32}$$

and

$$r = r_1. \tag{33}$$

This constitutes a system of 6 equations and 6 unknowns, which we can solve to compute the integral between 0 and t_a.

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